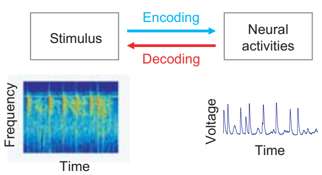
Linear Decodability for High-level Auditory Cortical Representations

Hiroki Asari[*1], Michael S. Wehr[*2], and Anthony M. Zádor[*2]

*1: Cold Spring Harbor Laboratory, Watson School of Biological Sciences, Cold Spring Harbor, 1 Bungtown Road, Cold Spring Harbor, NY 11724
*2: Cold Spring Harbor Laboratory, 1 Bungtown Road, Cold Spring Harbor, NY 11724
asari@cshl.edu   http://zadorlab.cshl.edu

Encoding vs. Decoding

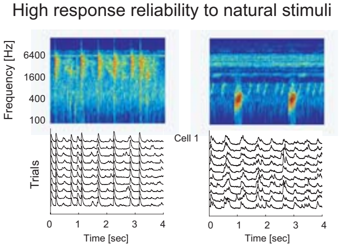
How to study the sensory processing in the brain?



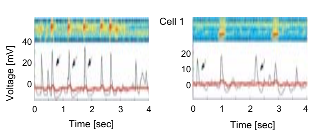
Question

Which performance is better,
linear encoding or linear decoding ?

High response reliability to natural stimuli

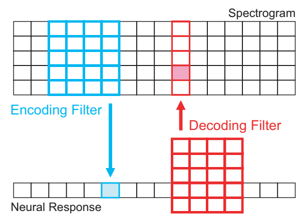


Linear encoding fails to predict neural response to natural stimuli.

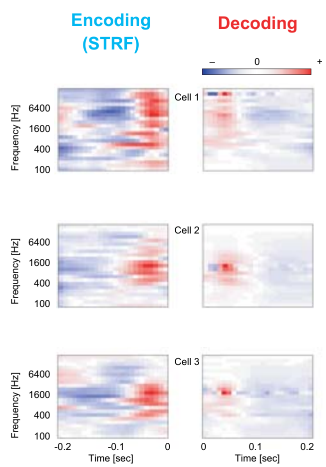


Machens, et al. (2004) J Neurosci.

Linear Filters

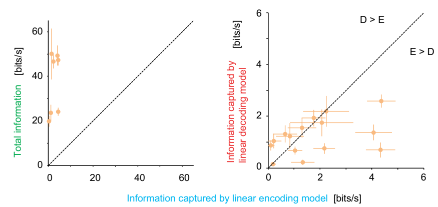


Examples of filters

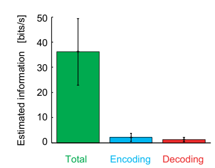


Information estimates

Total ≫ Linear encoding ≈ Linear decoding



Linear model captures little information in both encoding and decoding directions.

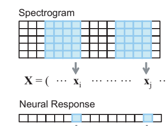


Summary

- Information captured by linear encoding model (STRF) is far less than total information.
- The performances of linear encoding and decoding are about equal.

Methods

Linear Regression



$\mathbf{X} = (\cdots \ \mathbf{x}_i \ \cdots\cdots\cdots \ \mathbf{x}_j \ \cdots)$

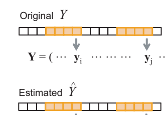
$\mathbf{y} = (\cdots \ y_i \ \cdots\cdots\cdots \ y_j \ \cdots)$

→ Encoding filter (λ : regularizer) $\beta_{ridge} = \mathbf{y}\mathbf{X}^T (\mathbf{X}\mathbf{X}^T + \lambda \mathbf{I})^{-1}$

Estimated response $\hat{\mathbf{y}}_{ridge} = {}_{ridge}\mathbf{X}$

Information Estimation (under Gaussian assumption)

$$I(Y,\hat{Y}) = H(Y) - H(Y|\hat{Y})$$
$$= H(Y) - H(N) \quad \text{(additive Gaussian noise)}$$
$$\approx \frac{1}{2}\log_2 \frac{\det \mathbf{A}_Y}{\det \mathbf{A}_N}$$



$\mathbf{Y} = (\cdots \ \mathbf{y}_i \ \cdots\cdots\cdots \ \mathbf{y}_j \ \cdots)$

$\hat{\mathbf{Y}} = (\cdots \ \mathbf{y}_i \ \cdots\cdots\cdots \ \mathbf{y}_j \ \cdots)$

→ Noise $\mathbf{N} = \mathbf{Y} - \hat{\mathbf{Y}}$

Covariance $\mathbf{A}_Y = \mathbf{Y}\mathbf{Y}^T = \mathbf{U}\mathbf{S}^2\mathbf{U}^T$

where $\mathbf{Y} = \mathbf{U}\mathbf{S}\mathbf{V}^T$ (Singular Value Decomposition)

Information captured by linear (encoding) model

Y : Neural response to various stimuli
$\hat{Y}$: The best linear estimate from recording data

Total Information (direct method)

Y : Neural response to the same stimulus
$\hat{Y}$: Deterministic part of the response (average)

Cf. Analysis in the Fourier domain

$$H(N) \sim \log_2 \sqrt{\det \mathbf{A}_N} = \frac{1}{2}\sum_{\omega} \log_2 P_N(\omega) \quad \text{(Power spectral density)}$$
$$H(Y) \sim \log_2 \sqrt{\det \mathbf{A}_Y} = \frac{1}{2}\sum_{\omega} \log_2 P_Y(\omega)$$
$$= \frac{1}{2}\sum_{\omega} \log_2 \left(P_N(\omega) + P_{\hat{Y}}(\omega)\right)$$

$$\rightarrow \quad I(Y,\hat{Y}) \approx \frac{1}{2}\log_2 \frac{\det \mathbf{A}_Y}{\det \mathbf{A}_N} = \frac{1}{2}\sum \left(1 + \mathrm{SNR}(\omega)\right)$$

where $\mathrm{SNR}(\omega) = \frac{P_{\hat{Y}}(\omega)}{P_N(\omega)}$ (Signal-to-noise ratio)