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Sparse Coding Predicts Noiseless Sensory Representations and Noisy Neurons

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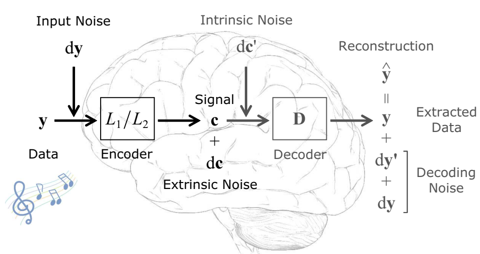
Abstract

The role of sparse overcomplete models of sensory representations has recently attracted attention on theoretical (e.g., Lewicki, 2002; Olshausen & Field, 2004; Asari et al., 2006) and experimental (e.g., Hromadka et al., 2008) grounds. Here we show that a natural consequence of a sparse overcomplete model is that ***represenations are noiseless at the population level even though single neuron responses can be highly variable.***

Noise Model

Observed Variability = Extrinsic Noise + Intrinsic Noise

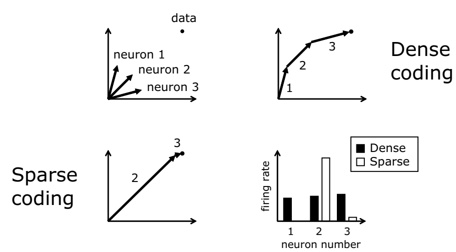


Decoding: $\mathbf{y} = \mathbf{D}\mathbf{c} = \sum_{i=1}^{M} c_i \mathbf{d}_i \quad \cdots \quad$ Eq.(1)

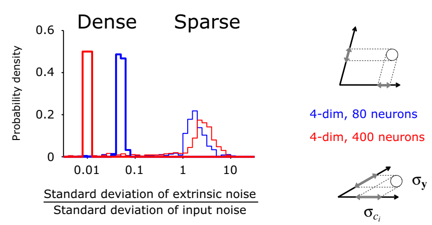
Sparse Encoding: $\min \|\mathbf{c}\|_1 = \sum_i |c_i|$ subject to Eq.(1)

Dense Encoding: $\min \|\mathbf{c}\|_2^2 = \sum_i c_i^2$ subject to Eq.(1)

Sparse vs. Dense Coding

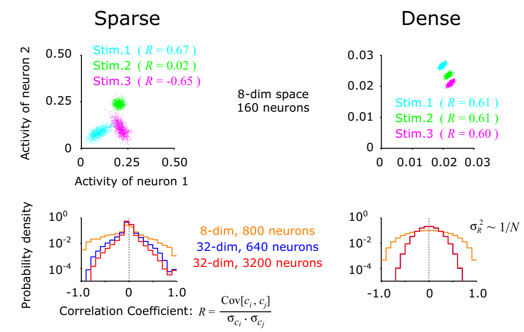


Single-cell Variability

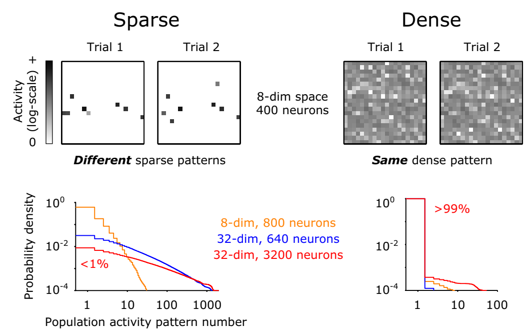


$$\left\langle \frac{\sigma_{c_i}}{\sigma_y} \right\rangle_i \sim \begin{cases} \sqrt[N-1]{M} & \text{(sparse)} \\ N/M & \text{(dense)} \end{cases} \quad \text{for } \{i \mid p(c_i + dc_i > 0) = 1\} \text{ and } N\text{-dim space} < M \text{ neurons}$$

Pairwise Variability



Population Variability



Summary

	Sparse	Dense
Objective function	L_1-norm	L_2-norm
Stimulus-dependence	Yes	No
Single-cell variability	High	Low
Pairwise variability	Laplacian	Gaussian
Population variability	High	Low
Intrinsic noise	Low(?)	High

Simulation Procedures

1. Generate M random overcomplete features $\mathbf{d}_i$ of unit length in N dimensional space.
2. Generate 100 random inputs $\mathbf{y}$ of unit length, each with additive i.i.d. noise (10,000 trials): $\mathbf{dy} \sim \mathrm{Gaussian}[0, (\|\mathbf{y}\|/100)^2]$.
3. Find L_1 and L_2 solutions: $\mathbf{y} + \mathbf{dy} \rightarrow \mathbf{c} + \mathbf{dc}$.
4. Analyze extrinsic variability at
   - Single-cell level: σ_y vs. σ_{c_i}
   - Pairwise level: dc_i vs. dc_j
   - Population level: $\{i \mid c_i + dc_i > \varepsilon = 10^{-8}\}$